MATHEMATICAL MODELLING OF CONVECTIVE FLOW OVER A HORIZONTAL CIRCULAR CYLINDER WITH CONVECTIVE BOUNDARY CONDITIONS IN VISCOUS, MICROPOLAR AND NANOFLUID

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I hereby declare that I have checked this thesis and in my opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy.

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I hereby declare that the work in this thesis is based on my original work except for quotations and citation which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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If I were to have kids one day, this is for them. I want them to know that once, their mom had a dream like they do

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ABSTRAK

Aliran bendalir dan pemindahan haba merupakan faktor penting dalam proses industri, pembuatan dan aplikasi kejuruteraan. Oleh sebab itu, adalah sangat perlu untuk memodelkan sistem bagi meningkatkan proses aliran bendalir dan pemindahan haba di mana kualiti akhir bagi sesuatu produk adalah bergantung kepada aliran kinematik serta pemanasan dan penyejukan secara serentak. Walau bagaimanapun, permasalahan matematik untuk aliran bendalir dan pemindahan haba terutamanya bagi geometri berbentuk silinder bulat mengufuk adalah sangat rumit untuk diselesaikan disebabkan wujudnya persamaan tak linear berpasangan. Bagi mendapatkan persamaan tepat, ia memerlukan masa yang lama dan usaha yang banyak manakala untuk menyediakan alat eksperimen pula memerlukan kos yang tinggi. Dalam kes ini, teknik berangka membuka jalan dalam mendapatkan penyelesaian terbaik untuk menyelesaikan masalah. Oleh itu, persamaan menakluk bagi aliran bendalir dan pemindahan haba beserta dengan syarat sempadan diselesaikan secara berangka. Kebiasaannya apabila model aliran olakan dilaksanakan, kebanyakan penyelidik menggunakan suhu dinding malar atau fluks haba malar sebagai syarat sempadan. Walau bagaimanapun, syarat sempadan ini tidak cukup lengkap untuk menggambarkan keadaaan proses pemanasan bagi sesetengah keadaan dalam industri. Terdapat satu lagi jenis syarat sempadan yang telah diperkenalkan di mana permukaan bawah silinder dipanaskan dengan olakan dan ini dikenali sebagai syarat sempadan olakan. Dimotivasikan oleh syarat sempadan yang baharu ini, skema berangka yang dibangunkan dalam kajian ini diharap dapat dijadikan sebagai teori rujukan kepada penyelesaian tepat atau kepada kerja makmal di masa hadapan. Oleh itu, lima masalah berbeza bagi aliran bendalir dan pemindahan haba telah dipertimbangkan dalam kajian ini dengan mengambil kira syarat sempadan olakan sebagai pemanasan haba. Semua model matematik ini kemudiannya diterbitkan bagi kes aliran olakan paksaan, bebas dan juga campuran di atas silinder bulat mengufuk di dalam tiga jenis bendalir berbeza iaitu likat, mikrokutub dan juga nanobendalir. Persamaan menakluk pembezaan separa parabola yang menerangkan tentang aliran kemudiannya ditukar kepada penjelmaan ketakserupaan, dan kemudiannya diselesaikan secara berangka menggunakan kaedah teknik pembezaan terhingga yang stabil tanpa syarat dikenali sebagai kaedah kotak-Keller. Kod berangka dalam bentuk atur cara komputer dibina menggunakan perisian MATLAB. Penyelesaian berangka terdiri daripada profil halaju, suhu, isipadu pecahan nanozarah, geseran permukaan, dan perubahan haba bagi nilai parameter yang berbeza untuk keadaan fizikal bagi parameter olakan, olakan campuran, nombor Lewis, parameter poros dan juga nombor Prandtl. Didapati, bagi kesemua masalah yang dipertimbangkan, profil halaju dan suhu meningkat bagi peningkatan syarat sempadan olakan. Bagi kes nanobendalir, profil isipadu pecahan nanozarah turut meningkat jika syarat sempadan olakan meningkat. Begitu juga bagi setiap kenaikan parameter olakan, pekali geseran permukaan juga meningkat kecuali bagi kes nanobendalir, di mana jika parameter olakan menurun, ia menunjukkan penurunan bagi kedua-dua kes; Tiwari dan Das serta Buongiorno. Manakala, bagi pekali perubahan haba dan Nusselt number, dapat diperhatikan bahawa kesan parameter olakan meningkat secara signifikan. Kesimpulannya, dengan menggunakan syarat sempadan olakan terhadap silinder bulat mengufuk, didapati bahawa trend yang diperoleh bagi kes sempadan olakan adalah menyerupai kes suhu permukaan malar apabila nilai parameter sempadan olakan $\gamma \to \infty$.

ABSTRACT

Fluid flow and heat transfer play a significant factor in industrial processes, manufacturing and engineering applications. Therefore, a model is needed to enhance the process of fluid flow and heat transfer, as the final products are heavily reliant upon the kinematics of the flow and the simultaneous heating or cooling. However, the mathematical description of fluid flow and heat transfer specifically in geometry of horizontal circular cylinder are quite difficult to solve due to the nonlinearity existence and coupled equations. Indeed, obtaining an analytical solution requires additional effort and time meanwhile to setup an experiment is costly. In such a case, numerical methods provide means to solve the problem. Therefore, the governing equation of fluid flow and heat transfer together with the boundary conditions are solved numerically. Normally when modelling convection flow, many researchers applied constant wall temperature or constant heat flux in the boundary conditions. Nevertheless, these types of boundary conditions appear insufficient to adequately describe the heating process for some cases. Another type of boundary condition; where convection heats the bottom surface of the cylinder are applied in this study. This type of heating process is called convective boundary condition. Motivated by this newly type of boundary condition, the numerical scheme derived in this research is anticipated to provide a theoretical reference to other analytical solution or for future experimental work. Five different problems of fluid flow and heat transfer have been considered by incorporating convective boundary conditions as thermal heating. Accordingly, these mathematical models are then derived for steady laminar forced, free, and mixed convection boundary layer flows over a horizontal circular cylinder immersed in three types of fluid namely viscous, micropolar fluid and nanofluid. The governing parabolic partial differential equations describing the flow are transformed using non-similar transformation, which is then solved numerically using the unconditionally stable implicit finite difference scheme known as the Keller-box method. The numerical codes in the form of computer programmes are developed using MATLAB software. The numerical results obtained consists of velocity, temperature, nanoparticle volume fraction profiles, skin friction and heat transfer for various parameters of physical conditions such as convective, mixed convection, Lewis number, porosity parameters, as well as Prandtl number. It was observed that in all considered problems, the profiles of velocity and temperature profiles increase for increased values of convective boundary conditions. In the case of nanofluids, the values of nanoparticle volume fraction profile increases with with the increment on the values of convective boundary condition. Correspondingly, as the value of convective parameter increases, the skin friction coefficient increase as well except for nanofluid where the convective parameter decreased in both cases; Tiwari and Das, and Buongiorno. However for heat transfer coefficient and Nusselt number, it was observed that the effects of convective parameter has increased significantly. In conclusion, by applying the convective boundary condition over a horizontal circular cylinder, it is found that the trend of the solutions obtained for the convective boundary condition case is similar to the constant wall temperature case, especially when convective parameter $\gamma \to \infty$.

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LIST OF SYMBOLS

a	Radius of cylinder
A	Area of the object
A_1	Rivlin-Ericksen tensor
A_m	Acceleration
C	Nanoparticle volume fraction
Cu	Copper/Cuprum
C_f	Skin friction coefficient
C_p	Specific heat
D_B	Brownian motion
D_T	Thermophoretic diffusion
e_t	Total energy
e_{int}	Kinetic energy
F_s	Surface forces
F_b	Body forces
f	Dimensionless stream function
Gr	Grashof number
G	Angular velocity
g	Gravity vector
h	Heat transfer coefficient
I	Identity tensor
h_f	Heat transfer coefficient
J	Microinertia per unit mass
K_1	Micropolar parameter
K_2	Permeability of the porous medium
k	Thermal conductivity
k_{nf}	Thermal conductivity of the nanofluids
k_s	Thermal conductivity of the solid
L	Length of the cylinder
Le	Lewis number
Nu	Nusselt number
N_b	Brownian motion parameter

N_r	Buoyancy ratio parameter
N_t	Thermophoresis parameter
Pe	Peclet number
p	Pressure
Pr	Prandtl number
Pr_c	Critical value of Prandtl number
q	Heat transfer per unit time
q_m	Mass flux
q_w	Heat flux
Ra	Rayleigh number
Re	Reynolds number
$S(t)$	Control surface
Sh	Sherwood number
T	Temperature
T_f	Temperature of the hot fluid
T_w	Local temperature
T_∞	Fluid temperature
U_∞	Free stream velocity
u,v	Velocity component in x,y direction
u_e	Free stream velocity
V	Fluid filtration velocity
$V(t)$	Control volume

Greek Symbol

α	Thermal diffusivity
α_{nf}	Thermal diffusivity of nanofluids
α_m	Effective thermal diffusivity
β	Thermal expansion coefficient
β_f	Thermal expansion coefficient of the fluid
β_s	Thermal expansion coefficient of the solid

δ	Boundary layer thickness
ε	Porosity of the porous medium
γ	Convective parameter
γ_c	Critical value of convective parameter
κ	Vortex viscosity
λ	Mixed convection parameter
μ	Viscosity
μ_f	Viscosity of the fluid fraction
ν	Kinematic viscosity
ϕ	Nanoparticle volume fraction
ψ	Stream function
ρ	Fluid density
ρ_p	Density of particle
ρ_{nf}	Density of nanofluid
ρ_{cm}	Effective heat capacity
$\rho_{f\infty}$	Density of the base fluid
τ	Shear stress
τ_w	Skin friction coefficient
θ	Nondimensional temperature
σ	Shear stress
ξ	Rate of shear

Subscripts

c	Critical value
f	Fluid
w	Surface condition
∞	Ambient condition
nf	Nanofluid
p	Particle

LIST OF ABBREVIATIONS

BVP	Boundary value problem
CBC	Convective boundary conditions
CHF	Constant heat flux
CWT	Constant wall temperature
CWFEM	Control volume finite element method
HAM	Homotopy analysis method
KBM	Keller-box method
MHD	Magnetohydrodynamic
NH	Newtonian heating
ODE	Ordinary differential equations
PDE	Partial differential equations
FDM	Finite difference method
FEM	Finite element method
FVM	Finite volume method
RKF	Runge-Kutta Fehlberg

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